

Original Article

Annihilator Ideal Graph of a Lattice with Two Atoms and Its Unique Ideal

P. N. Tayade

Head, Department of Mathematics, Lewa Educational Union's,
Dr. Annasaheb G. D. Bendale Mahila Mahavidyalaya (Autonomous), Jalgaon (M.S), India

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Correspondence Address:

Uddhao N. Ingle
Department of Electronics, Smt.
G. G. Khadse College,
Muktainagar Dist. Jalgaon
Email:
pntagdbmm@gmail.com

Quick Response Code:



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Abstract :

Let L be finite lattice with two atoms and unique ideal B . I studied annihilator ideal graph of lattice L and its unique ideal B . Explored that annihilator ideal graph of unique ideal is subgraph and both are bipartite. For two lattices, annihilator ideal graphs of unique ideals are isomorphic.

Keywords: Lattice, Annihilator ideal graph, subgraph, atom, unique ideal.

Introduction

Associating a graph to an algebraic structure is the interest of researcher in this subject of research and has attracted considerable attention in this area. To find the relationship between algebra, graph theory and applications of each other the idea of zero divisor graph of commutative ring R with identity was introduced by I Beck in [3]. He defined $T_0(R)$, zero divisor graph of ring R , whose vertices are elements of R and two distinct vertices x and y are adjacent if and only if $xy=0$. Coloring of $T_0(R)$ was his concern. Beck conjectured that $\omega(R)=\chi(R)$; $\chi(R)$ is the chromatic number and $\omega(R)$ clique number of $T_0(R)$. This investigation was continued by Anderson and Naseer in [2]. They gave counter example of above conjecture. Anderson and Livingston in [1] associated a zero divisor graph to a commutative ring R with identity by different method. They defined $T(R)$, zero divisor graph of ring R with identity, whose vertices are nonzero zero divisors of a ring R and two distinct vertices x and y are adjacent if and only if $xy=0$. The authors have studied zero divisor graphs of rings and some algebraic structures, in [1,2,4,5]. Annihilating ideal graphs of different structures have studied in [12,13]. In [6] the authors studied zero divisor graph of a finite lattice. In [5], Deore and Tayade studied the zero divisor graphs of lattices and its unique ideals. Study of zero divisor graph gives idea to introduce a unique ideal related to number of atoms in the lattice. Aim was to find relation between graph theory and lattice theory. In [11], P N Tayade introduced class of two atom lattices. The authors studied annihilator ideal graph of a lattice, $AnnIG(L)$ in [10]. In this paper, L be finite, atomic lattice with two atoms. We know every ideal is sub lattice. I studied the annihilator ideal graph of a lattice L , $AnnIG(L)$ and annihilator ideal graph of unique sublattice B , $AnnIG(B)$. Here shown that $AnnIG(B) \subseteq AnnIG(L)$ and both are bipartite graphs.

1. Lattice Theory Preliminaries

We recall some definitions and theorems from [7, 9]. A non empty set L equipped with a partially ordered relation $\leq$ is said to be partially order set or poset and denoted by $(L, \leq)$. A poset is a lattice if $\text{Sup}\{a,b\}$ and $\text{Inf}\{a,b\}$ exists in L , for any $a, b \in L$. We write $\text{Sup}\{a,b\}=a \vee b$, $\text{Inf}\{a,b\}=a \wedge b$. A lattice is an algebra $(L, \wedge, \vee)$ if 1. $a \wedge a = a$, $a \vee a = a$, 2. $a \wedge b = b \wedge a$, $a \vee b = b \vee a$, 3. $a \wedge (b \wedge c) = (a \wedge b) \wedge c$; $a \vee (b \vee c) = (a \vee b) \vee c$, 4. $a \vee (a \wedge b) = a$, $a \wedge (a \vee b) = a$, for all $a, b, c \in L$. Define an order $\leq$ as, for a, b in L , $a \leq b$ if and only if $a \wedge b = a$. Then $(L, \leq)$ is a partially ordered set and any two elements have least upper bound (lub) and greatest lower bound (glb). Conversely, let P be a partially ordered set with $\text{glb}\{a, b\}$, $\text{lub}\{a, b\} \in P$. Define $a \wedge b = \text{glb}\{a, b\}$ and $a \vee b = \text{lub}\{a, b\}$ then $(P, \wedge, \vee)$ is lattice. If there exist elements say 0 and 1 in L such that $0 \leq x \leq 1$, for all $x \in L$ then lattice L is said to be a bounded lattice. For $a \in L$, if there exist $b \in L$ such that $a \wedge b = 0$, $a \vee b = 1$, then b is said to be complement of a . Each element in lattice have complement, lattice is complemented.

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A bounded lattice is uniquely complemented if each element has a unique complement. If b, c are elements in L such that $b < x < c$ for no x in L then c is cover of b and denoted by $b \prec c$. If an element $(0 \neq) a \in L$ covers 0 then it is atom. For $(0 \neq) x \in L$, if there exist an atom a , such that $a \leq x$ then L is atomic lattice. A finite lattice with least element is atomic. A lattice L is modular iff for all $a \geq b$, $a \wedge (b \vee c) = b \vee (a \wedge c)$; $a, b, c \in L$. For distributive lattice $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$ for all $a, b, c \in L$.

A non empty subset I of L is ideal of lattice L , if $i \vee j \in I$ and $k \wedge i \in I$, for $i, j \in I, k \in L$. For $a \in L$, $(a] = \{k \in L \mid k \leq a\}$ is principal ideal generated by element a . For $A = \{a_1, a_2, a_3, \dots, a_n\} \subseteq L$ and $I = \{k \in L \mid k \leq a_1 \vee a_2 \vee a_3 \vee \dots \vee a_n\}$ then I is ideal L . An element $a \in L$ is zero divisor if there exist $(0 \neq) b \in L$ such that $a \wedge b = 0$. The set of all zero divisors in L is denoted by $Z(L)$. The set of non-zero zero divisors is $Z(L)^* = \{a \in L \mid a \neq 0, \exists 0 \neq b \in L \text{ such that } a \wedge b = 0\}$. The set of atoms in L is $A(L)$ and $A(I) = I \cap A(L)$ is the set of atoms in an ideal I .

2. Graph Theory Preliminaries

Let V be a non empty set of vertices and set of edges E is a set of 2- subsets of V then graph $G = (V, E)$ is undirected graph on vertex set V and edge set E . The 2 subset $e = \{u, w\} \in E$ is an edge between vertices u and w , in this case we say u and w are adjacent in G . For two distinct vertices x and y if there exist finite number of distinct vertices $a = u_1, u_2, u_3, \dots, u_n = b$ such that each pair of vertices $\{u_k, u_{k+1}\}, 1 \leq k \leq n-1$ is an edge then such finite sequence of vertices is path from vertex a to b . The number of edges in shortest path is its length. The length of shortest path from a to b is denoted by $d(a, b)$. The $\text{diam}(G) = \sup\{d(a, b) : a, b \in V\}$. The closed path is cycle. Girth of a graph G is the length of shortest cycle in G and it is denoted by $\text{gr}(G)$. If there is a path between any two vertices of a graph G , then G is said to be connected otherwise, disconnected. If there exist subsets $V_1, V_2, V_3, \dots, V_n$, of V such that $V_i \cap V_k = \emptyset, i \neq k, \cup V_i = V$ and no two vertices in same V_k are adjacent and any two vertices from different V_k are adjacent then the graph G is complete n - partite graph, $K(|V_1|, |V_2|, \dots, |V_n|)$. Any pair of vertices in a graph G is connected by an edge then graph G is complete graph. Complete graph on n - number of vertices is denoted by K_n . A undirected graph without loop and parallel edges is simple graph.

Two simple graphs $G = (V, E)$ and $H = (U, K)$ are isomorphic if there exist a one one onto mapping $f: V \rightarrow U$ such that $\{a, b\}$ is an edge in E if and only if $\{f(a), f(b)\}$ is an edge in K . The mapping f is known as graph isomorphism. We write it as $G \cong H$. In [8] the authors given good introduction to graph theory.

3. AnnIG(L) AND AnnIG(B)

Theorem 3.1. ([5], Theorem 2) Let L be a non diamond lattice with the least element 0 and having a finite set of atoms. Then there exist a finite, distributive and uniquely complemented ideal B of L such that $A(L) = A(B)$.

Corollary 3.2. A unique ideal B of lattice L is a sublattice.

Lemma 3.3. Let L be lattice and $a \in L$ then $(a] = \{x \in L \mid x \leq a\}$ is ideal of L .

Proof. For $y, z \in (a], y \leq a, z \leq a$, we have $y \vee z \leq a$ and for $k \in L, z \wedge k \leq a \wedge k \leq a$, implies $y \wedge k \in (a]$. Ideal $(a]$ is called principal ideal generated by a . Each ideal is a sublattice. In next lemma we treat ideal B as sublattice of lattice L .

Lemma 3.4. Ideal I of unique ideal B is ideal of lattice L .

Proof. By Theorem 3.1, Let $B = (avb], I = (x]$, for some $x \in B (\subseteq L)$, hence by Lemma 3.4, I is ideal of L .

Definition 3.5. ([10], Definition 1.1) Let L be lattice and $N(L) = \{I \mid I \text{ is ideal of } L : \text{Ann}(I) \neq \{0\}\}$. The annihilator ideal graph $\text{AnnIG}(L)$ of lattice L is the graph with vertex set $N(L)$ and two vertices I and J in $N(L)$ are adjacent if and only if $I \cap \text{Ann}(J) \neq \{0\}$ or $\text{Ann}(I) \cap J \neq \{0\}$.

Definition 3.6. Let B is unique ideal of lattice L . Let $N(B) = \{C \mid C \text{ is ideal of } B : \text{Ann}(C) \neq \{0\}\}$. The annihilator ideal graph $\text{AnnIG}(B)$ of sublattice B is a graph with vertex set $N(B)$ and two distinct vertices C and D in $N(B)$ are adjacent if and only if $C \cap \text{Ann}(D) \neq \{0\}$ or $\text{Ann}(C) \cap D \neq \{0\}$.

Lemma 3.7. ([10], Lemma 2.2). If L be a lattice then for any $I, J \in N(L)$, I and J are adjacent in $\text{AnnIG}(L)$ if and only if $A(I) \neq A(J)$.

Lemma 3.8 For finite lattice L , with unique ideal B and $A(L) = \{a, b\}$ then $N(B) \subseteq N(L)$.

Proof. $(a], (b] \in N(B)$ and

by Lemma 3.7, $A((a]) \neq A((b])$. By Lemma 3.4, $(a], (b] \in N(L)$

Theorem 3.9. Let L be a finite lattice then $\text{AnnIG}(L)$ is complete bipartite graph.

Proof. Let L be a lattice with $A(L) = \{a, b\}$.

Define $U = \{(x] \in N(L) : \text{unique } a \in (x]\}$ then $(a] \in U$ and $W = \{(y] \in N(L) : \text{unique } b \in (y]\}$, $(b] \in W$. For $(x], (z] \in U, A((x]) = A((z])$, implies ideals $(x]$ and $(z]$ are not adjacent in $\text{AnnIG}(L)$. Similarly $(p], (q] \in W, A((p]) = A((q])$, implies ideals $(p]$ and $(q]$ are not adjacent in $\text{AnnIG}(L)$. For $(x] \in U, (p] \in W, A((x]) \neq A((p])$, ideals $(x]$ and $(p]$ are adjacent in $\text{AnnIG}(L)$. Again $U \cap W = \emptyset$ and $U \cup W = N(L)$, proves that $\text{AnnIG}(L)$ is complete bipartite.

Corollary 3.10. Let L be a lattice and B is its unique ideal then $\text{AnnIG}(B)$ is bipartite graph $K_{1,1}$.

Proof. Here $A(B) = A(B) \cap A(L) = \{a, b\}$ and $\{(a] \} \subseteq U, \{(b] \} \subseteq W$.

$A((a]) \neq A((b])$. Hence $\text{AnnIG}(B) = K_{1,1}$.

Corollary 3.11. Let L be a lattice then following statements hold.

1. If $|U| = 1 = |W|$, then $\text{AnnIG}(B) = \text{AnnIG}(L)$ is bipartite graph $K_{1,1}$.

2. If $|U| = 1, |W| = n$, then $\text{AnnIG}(L)$ is star graph $K_{1,n}$ and

$\text{AnnIG}(B) = K_{1,1}$

3.If $|U|=2, |W| \geq n, n \geq 2$ then $AnnIG(L)$ is complete bipartite graph $K_{2,n}$ and $AnnIG(B)=K_{1,1}$

$A(\overline{a}) \neq A(\overline{b})$. Hence $AnnIG(B)=K_{1,1}$.

Proposition 3.12. Let B be unique ideal of lattice L then $AnnIG(B) \subseteq AnnIG(L)$.

Proof. By definition 3.5 and 3.6, $N(B) \subseteq N(L)$. We have $(\overline{a}), (\overline{b}) \in N(B)$ and

$A(\overline{a}) \neq A(\overline{b})$, hence $(\overline{a})$ and $(\overline{b})$ are adjacent in $AnnIG(B)$. By definition of subgraph, $AnnIG(B) \subseteq AnnIG(L)$.

Let $L = (L, \leq)$ and $F = (F, \leq)$ be two lattices. A one onto map $g: L \rightarrow F$ is called an lattice isomorphism if $x \leq y \Leftrightarrow g(x) \leq g(y)$ and we write it as $L \cong F$

4 Examples

Example 4.1. Let $L = (D_{10}, |)$ and $M = (D_{12}, |)$ are two lattices with divisibility order and have unique ideals B and C respectively then

1. $AnnIG(B) \cong AnnIG(C)$.

2. $gr(AnnIG(B)) = gr(AnnIG(C)) = \infty$

3. $AnnIG(L) \subset AnnIG(M)$.

4. $gr(AnnIG(L)) = gr(AnnIG(M)) = \infty$

Example 4.2. Let $L = (D_{21}, |)$ and $K = (D_{36}, |)$ be two lattices with divisibility order and unique ideals B and C respectively. Then

1. $AnnIG(B) \cong AnnIG(C)$.

2. $gr(AnnIG(B)) = gr(AnnIG(C)) = \infty$

3. $AnnIG(L) \subset AnnIG(K)$.

4. $gr(AnnIG(L)) \neq gr(AnnIG(K))$.

Conclusions

For two lattices if unique ideals exist, annihilator ideal graphs of unique ideals are isomorphic. Annihilator ideal graph of Boolean lattice and its unique ideal are same.

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Conflicts of interest

The authors declare that there are no conflicts of interest regarding the publication of this paper

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