

Original Article

On Semi n -Absorbing Subsemimodules

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Abstract:

Let R be a commutative semiring with identity and M an R -semimodule. In this paper, we introduce and study the notion of semi- n -absorbing subsemimodules, which extends the concept of semi- n -absorbing submodules from module theory to the broader setting of semimodules over semirings. We obtain several fundamental characterizations of semi- n -absorbing subsemimodules in terms of annihilators, cyclic subsemimodules, and colon constructions. Their behavior is investigated under homomorphisms, intersections, and quotient semimodules, along with their stability under direct sums. A complete classification of semi- n -absorbing ideals of the semiring $\mathbb{Z}_0^+$ is also provided. These results generalize and unify earlier works on n -absorbing and semi- n -absorbing submodules, contributing to the structural theory of semimodules over commutative semirings.

Keywords: Semi n -absorbing subsemimodule; Q -subsemimodule; subtractive subsemimodule; semiring.

Introduction

The theory of semirings and semimodules offers a natural generalization of classical ring and module theory. It has garnered significant attention due to its applications in algebra, theoretical computer science, automata theory, and optimization. A comprehensive treatment of semirings and their semimodules can be found in the monograph of Golan [5]. Throughout this paper, all semirings are assumed to be commutative with a nonzero identity, and all semimodules are left semimodules. Unlike modules over rings, semimodules over semirings lack additive inverses, leading to several structural and technical differences. Consequently, concepts such as subtractive subsemimodules, partitioning (or Q -) subsemimodules, and annihilators play a crucial role in developing a meaningful ideal and subsemimodule theory. These notions were independently introduced and studied by Atani and Atani [1], and by Chaudhari and Bonde [2], who established foundational results concerning partitioning subsemimodules and quotient constructions. Further developments on direct sums and exact sequences of semimodules can be found in [3,4]. In module theory over rings, various generalizations of prime submodules have been introduced. Among these, the notion of n -absorbing submodules was introduced by Darani and Soheilnia [6], extending the classical concept of prime submodules. Subsequently, Khoshdel and Maani-Shirazi [7] introduced semi- n -absorbing submodules, which weaken the defining conditions. Motivated by these developments, the primary purpose of this paper is to introduce and investigate the concept of semi- n -absorbing subsemimodules of semimodules over commutative semirings. We establish several equivalent characterizations of these subsemimodules using annihilators and cyclic subsemimodules. Their stability under homomorphisms, intersections, and quotient constructions is examined. Special attention is given to the semiring $\mathbb{Z}_0^+$, for which we provide a complete classification of semi- n -absorbing ideals. For the definitions of semiring and left semimodule over a semiring, we refer to [5]. We denote the sets of non-negative integers, positive integers, and non-negative rationals by $\mathbb{Z}_0^+$, $\mathbb{N}$, and $\mathbb{Q}_0^+$, respectively. The set $\mathbb{Z}_0^+$ forms a semiring under the usual addition and multiplication, but it is not a ring. For $a_1, a_2, \dots, a_n \in \mathbb{Z}_0^+$, we denote by $\langle a_1, a_2, \dots, a_n \rangle$ the ideal generated by these elements in the semiring $(\mathbb{Z}_0^+, +, \cdot)$. For definitions of left R -semimodules, subsemimodules, partitioning subsemimodules (i.e., Q -subsemimodules), subtractive subsemimodules (i.e., k -subsemimodules), quotient semimodules, semimodule homomorphisms, and direct sums, we refer to [1,2,3,5]. Throughout this paper, an R -semimodule means a left semimodule over a semiring R . An R -semimodule M is called zerosumfree if $x + y = 0$ implies $x = y = 0$ for all $x, y \in M$.

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An on to R -semimodule homomorphism $f: M \rightarrow M'$ is called maximal if for each $a \in M'$, there exists a unique $q_a \in f^{-1}(\{a\})$ such that $x + \ker f \subseteq q_a + \ker f$ for every $x \in f^{-1}(\{a\})$, where $\ker f = \{x \in M: f(x) = 0_{M'}\}$. A subsemimodule N of an R -semimodule M is called cyclic if $N = \{rx: r \in R\}$ for some $x \in M$. It is denoted by Rx or $\langle x \rangle$.

Lemma 1.1. (9) Let I and J be subtractive ideals of a semiring R . Then $I \cup J$ is a subtractive ideal of R if and only if $I \subseteq J$ or $J \subseteq I$.

Lemma 1.2. (1) If N is a subtractive subsemimodule of an R -semimodule M and $m \in M$, then:

1. $(N: M)$ is a subtractive ideal of R ;
2. $(N: m) = \{r \in R: rm \in N\}$ is a subtractive ideal of R ;
3. $(0: m) = \{r \in R: rm = 0\}$ is a subtractive ideal of R ;
4. $(0: M) = \{r \in R: rm = 0 \text{ for all } m \in M\}$ is a subtractive ideal of R .

A proper subsemimodule N of M is called prime if for $r \in R$ and $m \in M$, $rm \in N$ implies $r \in (N: M)$ or $m \in N$.

Definition 1.3 (2). A subsemimodule N of M is called partitioning (or a Q -subsemimodule) if there exists a subset $Q \subseteq M$ such that:

- (i) $M = \bigcup \{q + N: q \in Q\}$;
- (ii) For $q_1, q_2 \in Q$, $(q_1 + N) \cap (q_2 + N) \neq \emptyset$ if and only if $q_1 = q_2$.

If N is a partitioning subsemimodule of an R -semimodule M , then $M/N_{(Q)} = \{q + N: q \in Q\}$ forms an R -semimodule under the operations defined in [2], called the quotient semimodule.

Lemma 1.4. (2) Let N be a subsemimodule of an R -semimodule M and $x, y \in M$ such that $x + N \subseteq y + N$. Then for all $z \in M$ and $r \in R$, $x + z + N \subseteq y + z + N$ and $rx + N \subseteq ry + N$.

Lemma 1.5. (2) Let N be a Q -subsemimodule of an R -semimodule M . For each $x \in M$, there exists a unique $q \in Q$ such that $x + N \subseteq q + N$. Hence $x = q + a$ for some $a \in N$.

Semi- n -Absorbing Subsemimodules

Definition 2.6. Let $n > 1$ be a fixed integer. A proper subsemimodule N of an R -semimodule M is called a semi- n -absorbing subsemimodule if for all $r \in R$ and $m \in M$,

$$r^n m \in N \Rightarrow rm \in N \text{ or } r^n \in (N: M).$$

Example 2.7. Let p be a prime number and let n be a positive integer greater than 1. The zero subsemimodule of the $\mathbb{Z}_0^+$ -semimodule $\mathbb{Z}_p^n$ is a semi- n -absorbing subsemimodule.

Theorem 2.8. Let R be a commutative semiring, M an R -semimodule, and N a semi- n -absorbing subsemimodule of M . Then for every $m \in M$, the annihilator

$$(N: M) = \{r \in R: rm \in N\} \text{ is a semi-}n\text{-absorbing ideal of } R.$$

Proof. Let $a, b \in R$ and suppose $a^n b \in (N: m)$. Then $a^n b m \in N$. Since N is semi- n -absorbing, either $abm \in N$ or $a^n \in (N: M)$. Hence either $ab \in (N: m)$ or $a^n \in (N: M)$. Therefore $(N: m)$ is a semi- n -absorbing ideal of R . $\square$

Lemma 2.9. Let R be a commutative semiring, M an R -semimodule, N a proper subsemimodule of M , and $n \geq 2$ a fixed integer. The following statements are equivalent:

- (i) N is a semi- n -absorbing subsemimodule of M ;
- (ii) For every $a \in R$ and every subsemimodule L of M , $a^n L \subseteq N$ implies $aL \subseteq N$ or $a^n \in (N: M)$.

Proof. (i) $\Rightarrow$ (ii). Assume N is semi- n -absorbing and let $a \in R$ and $L \subseteq M$ with $a^n L \subseteq N$. Take $x \in L$. Then $a^n x \in N$. Since N is semi- n -absorbing, either $ax \in N$ or $a^n \in (N: M)$. If $a^n \in (N: M)$, we are done. Otherwise, $ax \in N$ for every $x \in L$, so $aL \subseteq N$.

(ii) $\Rightarrow$ (i). Let $a \in R$ and $m \in M$ with $a^n m \in N$. Take $L = Rm$, the cyclic subsemimodule generated by m . Then $a^n L \subseteq N$. By (ii), either $aL \subseteq N$, which implies $am \in N$, or $a^n \in (N: M)$. Hence N is semi- n -absorbing. $\square$

Lemma 2.10. Let R be a commutative semiring, M an R -semimodule, and $n \geq 2$ a fixed integer. Let N and K be subsemimodules of M with $N \subseteq K$. If N is a semi- n -absorbing subsemimodule of M , then N is a semi- n -absorbing subsemimodule of K .

Proof. This follows by adapting the argument from Lemma 2.9. Let $a \in R$ and $x \in K$ such that $a^n x \in N$. Since $K \subseteq M$, we have $x \in M$. Because N is semi- n -absorbing in M , either $ax \in N$ or $a^n \in (N: M)$. Since $N \subseteq K$, we have $(N: M) \subseteq (N: K)$. Hence $a^n \in (N: K)$. Therefore either $ax \in N$ or $a^n \in (N: K)$. Thus N is semi- n -absorbing in K . $\square$

Lemma 2.11. Let R be a commutative semiring, M an R -semimodule, and $n \geq 2$. Let N be a semi- n -absorbing subsemimodule of M . Then for every subsemimodule L of M , either $L \subseteq N$ or $L \cap N$ is a semi- n -absorbing subsemimodule of L .

Proof. Let L be a subsemimodule of M . If $L \subseteq N$, there is nothing to prove. Assume $L \not\subseteq N$. We show that $L \cap N$ is a semi- n -absorbing subsemimodule of L .

Let $a \in R$ and $x \in L$ such that $a^n x \in L \cap N$. Then $a^n x \in N$. Since N is semi- n -absorbing in M , either $ax \in N$ or $a^n \in (N: M)$.

If $ax \in N$, then $ax \in L \cap N$ because $x \in L$ and L is a subsemimodule. Hence $ax \in L \cap N$.

If $a^n \in (N: M)$, then $a^n M \subseteq N$. In particular, $a^n L \subseteq N$, so $a^n \in (L \cap N: L)$. Therefore, $L \cap N$ satisfies the semi- n -absorbing condition in L . $\square$

Lemma 2.12. Let R be a commutative semiring, M an R -semimodule, and $n \geq 2$. Let N be a proper subsemimodule of M . Then N is a semi- n -absorbing subsemimodule of M if and only if for every $a \in R$, we have

$$(N: M a^n) = (N: M a) \text{ or } (N: M a^n) = M.$$

Proof. Assume N is semi- n -absorbing and let $a \in R$. If $(N :_M a^n) = M$, there is nothing to prove. Suppose $(N :_M a^n) \neq M$. Let $x \in (N :_M a^n)$. Then $a^n x \in N$. Since N is semi- n -absorbing and $a^n \notin (N :_R M)$, it follows that $ax \in N$. Hence $x \in (N :_M a)$. Therefore $(N :_M a^n) \subseteq (N :_M a)$. The reverse inclusion always holds, so $(N :_M a^n) = (N :_M a)$.

Conversely, assume that for every $a \in R$, we have $(N :_M a^n) = (N :_M a)$ or $(N :_M a^n) = M$. Let $a \in R$ and $x \in M$ with $a^n x \in N$. Then $x \in (N :_M a^n)$. If $(N :_M a^n) = M$, then $a^n \in (N :_R M)$. Otherwise, $(N :_M a^n) = (N :_M a)$, so $x \in (N :_M a)$, which gives $ax \in N$. Thus N is semi- n -absorbing. $\square$

Lemma 2.13. *Let R be a commutative semiring, M an R -semimodule, and $n \geq 2$. Let N be a proper subsemimodule of M . Then the following are equivalent:*

- (i) N is a semi- n -absorbing subsemimodule of M ;
- (ii) $(N :_M I)$ is a semi- n -absorbing subsemimodule of M for every ideal I of R with $IM \not\subseteq N$;
- (iii) $(N :_M r)$ is a semi- n -absorbing subsemimodule of M for every $r \in R$ with $rM \not\subseteq N$.

Proof. (i) $\Rightarrow$ (ii). Let I be an ideal of R with $IM \not\subseteq N$. Let $a \in R$ and L be a subsemimodule of M such that $a^n L \subseteq (N :_M I)$. Then $a^n IL \subseteq N$. Since N is semi- n -absorbing, either $aIL \subseteq N$ or $a^n \in (N :_R M)$. If $aIL \subseteq N$, then $aL \subseteq (N :_M I)$. Hence $(N :_M I)$ is semi- n -absorbing.

(ii) $\Rightarrow$ (iii). This follows by taking $I = Rr$.

(iii) $\Rightarrow$ (i). Taking $r = 1$, we obtain $(N :_M 1) = N$, hence N is semi- n -absorbing. $\square$

Lemma 2.14. *Let R be a commutative semiring, M an R -semimodule, and $n \geq 2$. If N is a semi- n -absorbing subsemimodule of M , then $(N :_R Rx)$ is a semi- n -absorbing ideal of R for every $x \in M \setminus N$.*

Proof. Since $x \notin N$, we have $(N :_R Rx) \neq R$. Let $a, b \in R$ with $a^n b \in (N :_R Rx)$. Then $a^n bx \in N$. Since N is semi- n -absorbing, either $abx \in N$ or $a^n \in (N :_R M)$. Hence either $ab \in (N :_R Rx)$ or $a^n \in (N :_R Rx)$. Thus $(N :_R Rx)$ is a semi- n -absorbing ideal of R . $\square$

Proposition 2.15. *Let $f: M_1 \rightarrow M_2$ be an R -semimodule homomorphism. Then:*

- (i) If f is onto and steady, and N is a semi- n -absorbing subtractive subsemimodule of M_1 containing $\ker f$, then $f(N)$ is a semi- n -absorbing subsemimodule of M_2 .
- (ii) If K is a semi- n -absorbing subsemimodule of M_2 , then $f^{-1}(K)$ is a semi- n -absorbing subsemimodule of M_1 .

Proof. (i) Assume f is onto and steady, and N is a semi- n -absorbing subtractive subsemimodule of M_1 containing $\ker f$. Let $a \in R$ and $m_2 \in M_2$ be such that $a^n m_2 \in f(N)$. Since f is onto, there exist $m_1 \in M_1$ and $n \in N$ such that $m_2 = f(m_1)$ and $a^n m_2 = f(n)$. Then

$$f(n) = a^n m_2 = a^n f(m_1) = f(a^n m_1).$$

Since f is steady, there exist $k_1, k_2 \in \ker f$ such that $n + k_1 = a^n m_1 + k_2$. Now N is subtractive and $\ker f \subseteq N$, so $a^n m_1 \in N$. Since N is semi- n -absorbing, either $am_1 \in N$ or $a^n \in (N : M_1)$.

If $am_1 \in N$, then $am_2 = af(m_1) = f(am_1) \in f(N)$.

If $a^n \in (N : M_1)$, then for any $m'_2 \in M_2$, there exists $m'_1 \in M_1$ with $m'_2 = f(m'_1)$, and

$$a^n m'_2 = a^n f(m'_1) = f(a^n m'_1) \in f(N),$$

so $a^n \in (f(N) : M_2)$. Hence $f(N)$ is semi- n -absorbing in M_2 .

(ii) Assume K is semi- n -absorbing in M_2 . Let $a \in R$ and $m_1 \in M_1$ be such that $a^n m_1 \in f^{-1}(K)$. Then

$$f(a^n m_1) = a^n f(m_1) \in K.$$

Since K is semi- n -absorbing, either $af(m_1) \in K$ or $a^n \in (K : M_2)$.

If $af(m_1) \in K$, then $am_1 \in f^{-1}(K)$.

If $a^n \in (K : M_2)$, then for any $m'_1 \in M_1$, we have $(a^n)m'_1 \in f^{-1}(K)$, so $a^n \in (f^{-1}(K) : M_1)$. Hence $f^{-1}(K)$ is semi- n -absorbing in M_1 . $\square$

Proposition 2.16. *Let $n > 1$, M_1 and M_2 be R -semimodules, and $M = M_1 \oplus M_2$. Moreover, let N_1 and N_2 be proper subsemimodules of M_1 and M_2 , respectively. Then:*

- (i) N_1 is a semi- n -absorbing subsemimodule of M_1 if and only if $N_1 \oplus M_2$ is semi- n -absorbing in M .
- (ii) N_2 is a semi- n -absorbing subsemimodule of M_2 if and only if $M_1 \oplus N_2$ is semi- n -absorbing in M .

Proof. We prove (i); the proof of (ii) is analogous.

Suppose N_1 is semi- n -absorbing in M_1 and let $a \in R$, $(x_1, x_2) \in M$ be such that $a^n(x_1, x_2) \in N_1 \oplus M_2$. Then $a^n x_1 \in N_1$. By hypothesis, either $ax_1 \in N_1$ or $a^n \in (N_1 : M_1)$.

If $ax_1 \in N_1$, then $a(x_1, x_2) = (ax_1, ax_2) \in N_1 \oplus M_2$.

If $a^n \in (N_1 : M_1)$, then $a^n \in (N_1 \oplus M_2 : M)$. Hence $N_1 \oplus M_2$ is semi- n -absorbing in M .

Conversely, assume $N_1 \oplus M_2$ is semi- n -absorbing in M . Let $a \in R$ and $x_1 \in M_1$ be such that $a^n x_1 \in N_1$. Then $a^n(x_1, 0) \in N_1 \oplus M_2$. Since $N_1 \oplus M_2$ is semi- n -absorbing, either $a(x_1, 0) \in N_1 \oplus M_2$ or $a^n \in (N_1 \oplus M_2 : M)$. In the first case, $ax_1 \in N_1$. In the second case, $a^n \in (N_1 : M_1)$. Thus N_1 is semi- n -absorbing in M_1 . $\square$

Proposition 2.17. *Let $n > 1$, M_1 and M_2 be R -semimodules such that $a^n M_1 + a^n M_2 = R$, and let N be a semi- n -absorbing subsemimodule of $M = M_1 \oplus M_2$. Then one of the following holds:*

- (i) $N = N_1 \oplus M_2$ where N_1 is semi- n -absorbing in M_1 ;
- (ii) $N = M_1 \oplus N_2$ where N_2 is semi- n -absorbing in M_2 ;
- (iii) $N = N_1 \oplus N_2$, where N_1 and N_2 are semi- n -absorbing in M_1 and M_2 , respectively.

Proof. By standard arguments, we have $N = N_1 \oplus N_2$ for some subsemimodules $N_1 \subseteq M_1$ and $N_2 \subseteq M_2$. If $N = N_1 \oplus M_2$ or $N = M_1 \oplus N_2$, the result follows from Proposition 2.16. Otherwise, $N = N_1 \oplus N_2$, where N_1 and N_2 are proper subsemimodules of M_1 and M_2 , respectively.

Let $a \in R$ and $x_1 \in M_1$ be such that $a^n x_1 \in N_1$. Then $a^n(x_1, 0) \in N = N_1 \oplus N_2$. By hypothesis, either $a(x_1, 0) \in N_1 \oplus N_2$ or $a^n \in (N_1 \oplus N_2 : M)$. In the first case, $ax_1 \in N_1$. In the second case, $a^n \in (N_1 : M_1)$. Thus N_1 is semi- n -absorbing in M_1 . Similarly, N_2 is semi- n -absorbing in M_2 . $\square$

Semi- n -Absorbing Subsemimodules in Quotient Semimodules

Theorem 3.1. Let N be a Q -subsemimodule of an R -semimodule M and P a subtractive subsemimodule of M with $N \subseteq P$. Then P is a semi- n -absorbing subsemimodule of M if and only if $P/N_{(Q \cap P)}$ is a semi- n -absorbing subsemimodule of the quotient semimodule $M/N_{(Q)}$.

Proof. Let N be a Q -subsemimodule of M and P a subtractive subsemimodule of M with $N \subseteq P$. By a known result, $P/N_{(Q \cap P)}$ is a subtractive subsemimodule of $M/N_{(Q)}$.

($\Rightarrow$) Suppose P is semi- n -absorbing. Let $r_1, r_2, \dots, r_{n+1} \in R$ and $q_1 + N, q_2 + N, \dots, q_{n+1} + N \in M/N_{(Q)}$ such that

$$0_{M/N_{(Q)}} \neq r_1 \odot (q_1 + N) \oplus r_2 \odot (q_2 + N) \oplus \dots \oplus r_{n+1} \odot (q_{n+1} + N) \in P/N_{(Q \cap P)}.$$

By the definition of the quotient operations, there exists a unique $q_0 \in Q \cap P$ such that

$$r_1 q_1 \cdots r_{n+1} q_{n+1} + N \subseteq q_0 + N.$$

Hence we can write $r_1 q_1 \cdots r_{n+1} q_{n+1} = q_0 + n$ for some $n \in N \subseteq P$. Since $q_0 \in P$ and $n \in P$, we have $r_1 q_1 \cdots r_{n+1} q_{n+1} \in P$.

Now P is semi- n -absorbing, so there exists $i \in \{1, 2, \dots, n+1\}$ such that

$$r_1 q_1 \cdots \widehat{r_i q_i} \cdots r_{n+1} q_{n+1} \in P,$$

where $\widehat{r_i q_i}$ means the i -th term is omitted. Let

$$r_1 \odot (q_1 + N) \oplus \dots \oplus r_i \odot (\widehat{q_i} + N) \oplus \dots \oplus r_{n+1} \odot (q_{n+1} + N) = q' + N$$

where $q' \in Q$. Since $r_1 q_1 \cdots \widehat{r_i q_i} \cdots r_{n+1} q_{n+1} \in P$ and $N \subseteq P$, we have $q' \in Q \cap P$. Therefore

$$r_1 \odot (q_1 + N) \oplus \dots \oplus r_i \odot (\widehat{q_i} + N) \oplus \dots \oplus r_{n+1} \odot (q_{n+1} + N) \in P/N_{(Q \cap P)}.$$

Hence $P/N_{(Q \cap P)}$ is semi- n -absorbing in $M/N_{(Q)}$.

($\Leftarrow$) Conversely, suppose $P/N_{(Q \cap P)}$ is semi- n -absorbing in $M/N_{(Q)}$. Let $r_1, \dots, r_{n+1} \in R$ and $m_1, \dots, m_{n+1} \in M$ with

$$0_M \neq r_1 m_1 \cdots r_{n+1} m_{n+1} \in P.$$

Since N is a Q -subsemimodule of M , by Lemma 1.5, for each m_i there exists a unique $q_i \in Q$ such that $m_i + N \subseteq q_i + N$. Then

$$r_1 \odot (q_1 + N) \oplus \dots \oplus r_{n+1} \odot (q_{n+1} + N) \in P/N_{(Q \cap P)}$$

and it is non-zero in the quotient.

By the semi- n -absorbing property in the quotient, there exists i such that

$$r_1 \odot (q_1 + N) \oplus \dots \oplus r_i \odot (\widehat{q_i} + N) \oplus \dots \oplus r_{n+1} \odot (q_{n+1} + N) \in P/N_{(Q \cap P)}.$$

Let $q' \in Q \cap P$ be the unique element corresponding to this sum. Then by Lemma 1.5,

$$r_1 m_1 \cdots \widehat{r_i m_i} \cdots r_{n+1} m_{n+1} = q' + n \in P$$

for some $n \in N \subseteq P$. Since P is subtractive, we conclude

$$r_1 m_1 \cdots \widehat{r_i m_i} \cdots r_{n+1} m_{n+1} \in P.$$

Hence P is semi- n -absorbing in M . $\square$

Conclusion

In this paper, we extended the concept of semi- n -absorbing substructures from modules over rings to semimodules over semirings. By carefully reformulating classical definitions to accommodate the absence of additive inverses, we established foundational properties including:

1. Equivalent characterizations using annihilators and cyclic subsemimodules;
2. Behavior under homomorphisms, intersections, and quotient constructions;
3. Stability conditions for direct sums;
4. A complete classification over the semiring $\mathbb{Z}_0^+$.

These results demonstrate that semi- n -absorbing behavior naturally arises in semimodule theory and offers a robust framework for further investigation in semiring-based algebraic systems. Future work may explore connections to tropical geometry, optimization theory, and applications in computer science where semirings play a fundamental role.

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Conflicts of interest

The authors declare that there are no conflicts of interest regarding the publication of this paper

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