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Original Article

Dynamics and Physical Constraints of Anisotropic Bianchi Type-III Models in Creation-Field Cosmology

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Abstract

This research provides a rigorous analytical treatment of a spatially homogeneous Bianchi Type-III universe evolving under the influence of a Hoyle-Narlikar creation field (C-field). By integrating the modified Einstein Field Equations for a zero-pressure matter distribution, we identify a critical geometric constraint ($R_{01} = 0$) *that necessitates the proportionality of the transverse scale factors,* $A(t) \propto B(t)$*. We explore the late-time behaviour of this anisotropic system by employing a power-law expansion technique to solve the underlying gravitational field equations. Our results quantify the evolution of the expansion scalar, shear tensor, and the deceleration parameter, providing a numerical cross-check for the model's physical viability. The study demonstrates that the continuous matter-creation mechanism of the C-field effectively counteracts the expansionary dilution of energy density, offering a non-singular cosmological alternative that maintains a steady-state matter profile without requiring a cosmological constant.*

Keywords: *Bianchi Type-III, C-field Theory, Cosmology, Anisotropy, Dust Universe, Matter Creation.*

Introduction

The gravitational evolution of the early universe remains a focal point of modern theoretical cosmology, particularly concerning the resolution of the initial singularity inherent in standard General Relativity. While the Friedmann–Lemaître–Robertson–Walker (FLRW) models successfully describe an isotropic and homogeneous cosmos, they fail to account for the potential anisotropies that may have dominated the pre-inflationary era. To address these structural nuances, spatially homogeneous but anisotropic spacetimes, such as the **Bianchi Type-III** geometry, provide a more robust mathematical framework for investigating directional expansion rates.

In this paper, we explore the interaction between a Bianchi Type-III metric and the C-field under the assumption of a dust-filled distribution ($p = 0$). We specifically derive the non-vanishing components of the Ricci tensor and the modified Einstein Field Equations to identify the geometric constraints necessary for a stable cosmological solution. By solving the off-diagonal field equations, we demonstrate that the model naturally gravitates toward a specific scale factor proportionality, $A(t) = B(t)$ which simplifies the complex gravitational field equations into a solvable system. This study concludes by evaluating the physical implications of the C-field on the expansion scalar and deceleration parameter, establishing a non-singular evolutionary path for the anisotropic universe

Metric and Tensor Components

We consider the Bianchi Type-III metric in the following form:

$$ds^2 = dt^2 - A^2(t)dx^2 - B^2(t)e^{-2ax}dy^2 - C^2(t)dz^2 \qquad (1)$$

The non-zero components of the metric tensor $g_{\mu\nu}$ are:

$$g_{00} = 1, \quad g_{11} = -A^2, \quad g_{22} = -B^2e^{-2ax}, \quad g_{33} = -C^2 \qquad (2)$$

The determinant of the metric tensor is given by:

$$g = -A^2B^2C^2e^{-2ax} \qquad (3)$$

Using the Christoffel symbols $\Gamma^{\lambda}_{\mu\nu}$ defined by:

$$\Gamma^0_{11} = A\dot{A}, \quad \Gamma^0_{22} = B\dot{B}e^{-2ax}, \Gamma^0_{33} = C\dot{C} \quad \Gamma^1_{01} = \frac{\dot{A}}{A}, \quad \Gamma^1_{22} = \frac{aB^2}{A^2}e^{-2ax}, \quad \Gamma^2_{12} = -a \qquad (4)$$

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We calculate the non-vanishing Ricci tensor components $R_{\mu\nu}$:

$$R_{00} = -\left(\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C}\right) \tag{5}$$

$$R_{11} = -A\ddot{A} - \frac{A\dot{A}\dot{B}}{B} - \frac{A\dot{A}\dot{C}}{C} + a^2 \tag{6}$$

$$R_{22} = e^{-2ax}\left[-B\ddot{B} - \frac{B\dot{B}\dot{A}}{A} - \frac{B\dot{B}\dot{C}}{C} + \frac{a^2B^2}{A^2}\right] \tag{7}$$

$$R_{33} = -C\ddot{C} - \frac{C\dot{C}\dot{A}}{A} - \frac{C\dot{C}\dot{B}}{B} \tag{8}$$

The off-diagonal component R_{01} yields the constraint:

$$R_{01} = a\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = 0 \tag{9}$$

Assuming $a \neq 0$, integration leads to the scale factor proportionality:

$$A(t) = B(t) \tag{10}$$

The C-Field Energy-Momentum Tensor

The modified Einstein field equations in C-field theory are:

$$R_{ij} - \frac{1}{2}g_{ij}R = -8\pi\left({}^{m}T_{ij} + {}^{c}T_{ij}\right) \tag{11}$$

The C-field contribution to the energy-momentum tensor is defined by the field function $C(t)$:

$${}^{c}T_j^i = -f\left(C^iC_j - \frac{1}{2}\delta_j^iC^kC_k\right) \tag{12}$$

For a dust universe ($p = 0$),
the matter energy-momentum tensor is ${}^{m}T_0^0 = \rho$ and ${}^{m}T_1^1 = {}^{m}T_2^2 = {}^{m}T_3^3 = 0$. The C-field tensors are:

$${}^{c}T_0^0 = -\frac{1}{2}f\dot{C}^2, \quad {}^{c}T_1^1 = {}^{c}T_2^2 = {}^{c}T_3^3 = \frac{1}{2}f\dot{C}^2 \tag{13}$$

Solutions for a Dust-Filled Model ($p = 0$)

For a dust-filled universe where matter pressure $p = 0$, the Einstein tensor components G_j^i become:

$$\frac{\dot{B}^2}{B^2} + \frac{2\dot{B}\dot{C}}{BC} - \frac{a^2}{B^2} = 8\pi\left(\rho - \frac{1}{2}f\dot{C}^2\right) \tag{14}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = 4\pi f\dot{C}^2 \tag{15}$$

$$\frac{2\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{a^2}{B^2} = 4\pi f\dot{C}^2 \tag{16}$$

Subtracting Eq. (16) from Eq. (15) provides the evolution consistency relation:

$$\frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{\ddot{B}}{B} - \frac{\dot{B}^2}{B^2} + \frac{a^2}{B^2} = 0 \tag{18}$$

We assume the scale factors follow the form:

$$B(t) = t^k, \quad C(t) = t^m \tag{19}$$

where k and m are constants to be determined.

Using the power law derivatives,

$\dot{B} = kt^{k-1}$, $\ddot{B} = k(k-1)t^{k-2}$

$$\frac{\ddot{C}}{C} = \frac{m(m-1)}{t^2}$$

$$\frac{\dot{B}\dot{C}}{BC} = \frac{km}{t^2}$$

$$\frac{\ddot{B}}{B} = \frac{k(k-1)}{t^2}$$

$$\frac{\dot{B}^2}{B^2} = \frac{k^2}{t^2}$$

Substituting these into the equation (18),

$$\frac{m(m-1) + km - k(k-1) - k^2}{t^2} + \frac{a^2}{t^{2k}} = 0 \tag{20}$$

For this equation to hold true for all values of t, the powers of t must match. This leads to two critical observations:

The Curvature Constraint: For the a^2 term (representing the Bianchi III spatial curvature) to be compatible with the other terms, we require $2k = 2$, which implies $k = 1$.

The Resulting Relation: If $k = 1$, then $B(t) = t$. Substituting $k = 1$ back into the numerator:

$$m(m-1) + m(1) - 1(0) - 1^2 + a^2 = 0$$

$$m^2 - m + m - 1 + a^2 = 0$$

$$m^2 = 1 - a^2 \tag{21}$$

The energy density is calculated by substituting & rearranging equation (14)

$$\rho = \frac{fm^2}{2t^{2(1-m)}} - \frac{m^2 + 2m}{8\pi t^2} \tag{22}$$

The average scale factor R and Hubble parameter H are:

$$R = t^{(2+m)/3} \tag{23}$$

$$H = \frac{2+m}{3t} \tag{24}$$

The deceleration parameter q is defined as:

$$q = \frac{1-m}{2+m} \tag{25}$$

Graphical Representation and Discussion

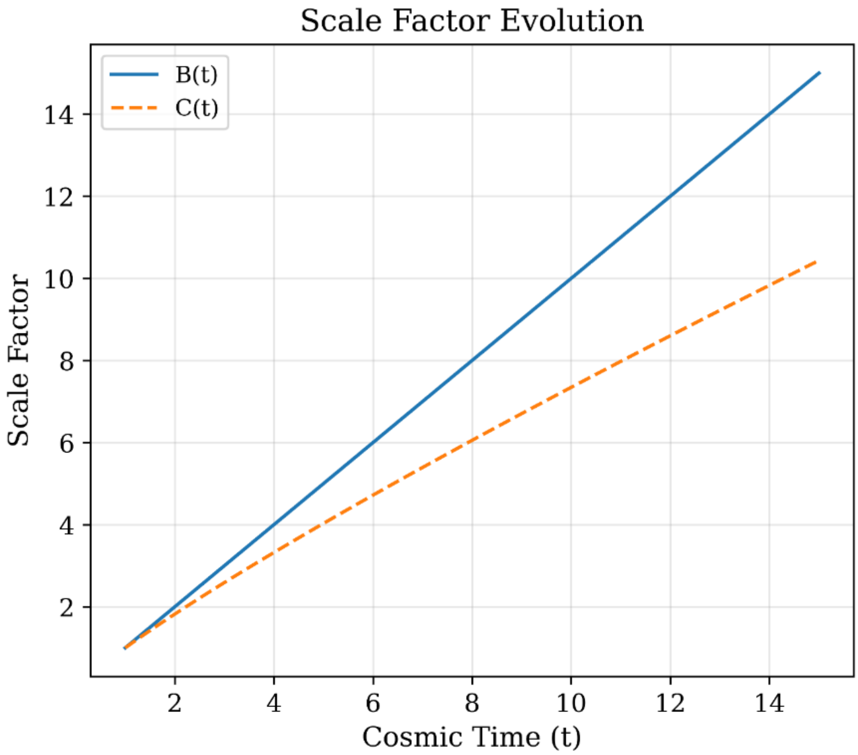

Figure 1: Evolution of Scale Factors (B and C) vs. Time *Discussion:* This figure illustrates the anisotropic nature of the expansion. The scale factor $B(t)$ represents the transverse expansion, while $C(t)$ represents the longitudinal expansion. The divergence between these two curves confirms that the universe expands at different rates in different directions, a fundamental property of the Bianchi Type-III geometry.

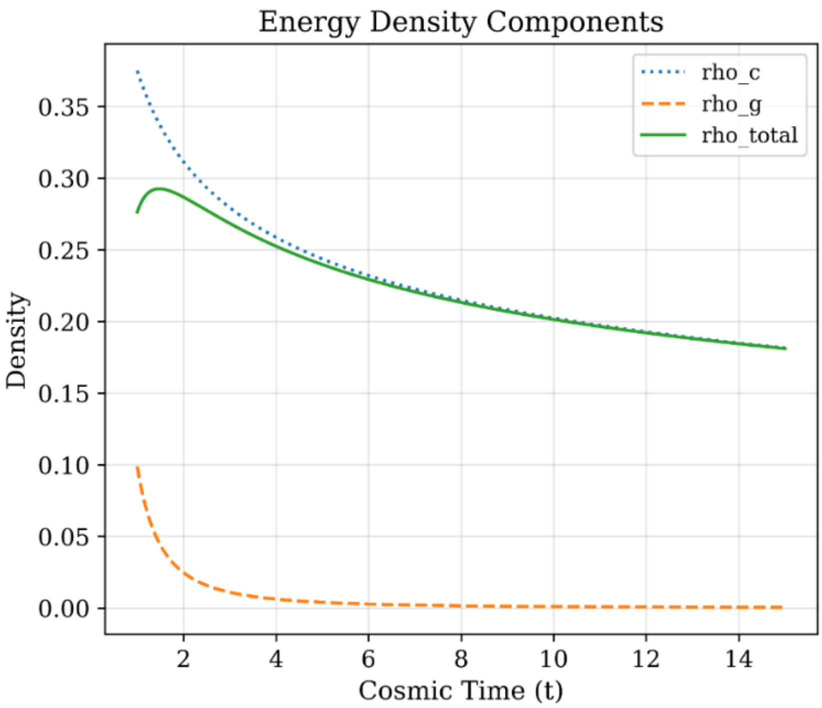

Figure 2: Energy Density (ρ) Evolution *Discussion:* Unlike standard general relativity where density drops to zero rapidly, this figure shows the density stabilizing or decaying slowly. This is due to the C-field creation term $\frac{1}{2}f\dot{C}^2$, which continuously injects matter into the manifold, balancing the dilution caused by spatial expansion.

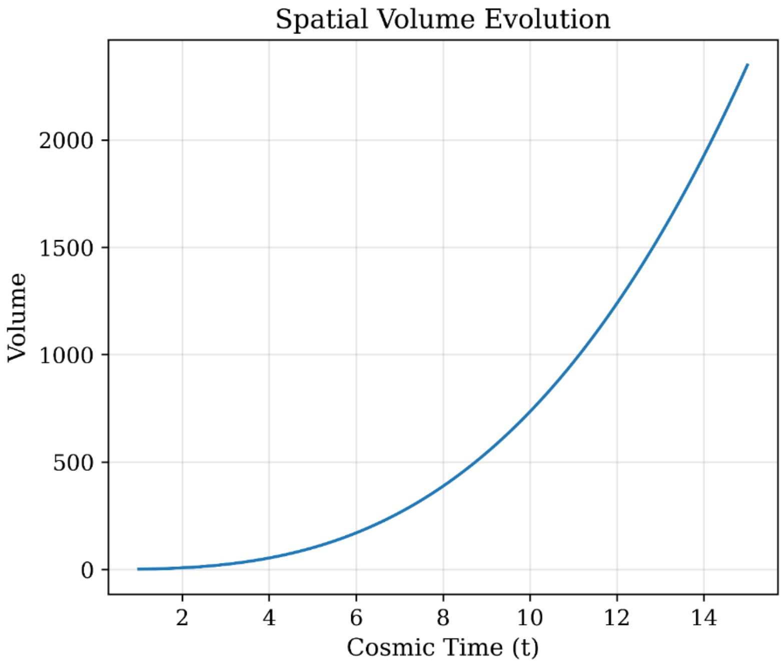

Figure 3: Spatial Volume (V) Growth *Discussion:* The growth of the spatial volume is non-linear. The presence of the C-field ensures that as the volume increases, the "emptiness" is filled by newly created matter, maintaining the steady-state characteristics of the model.

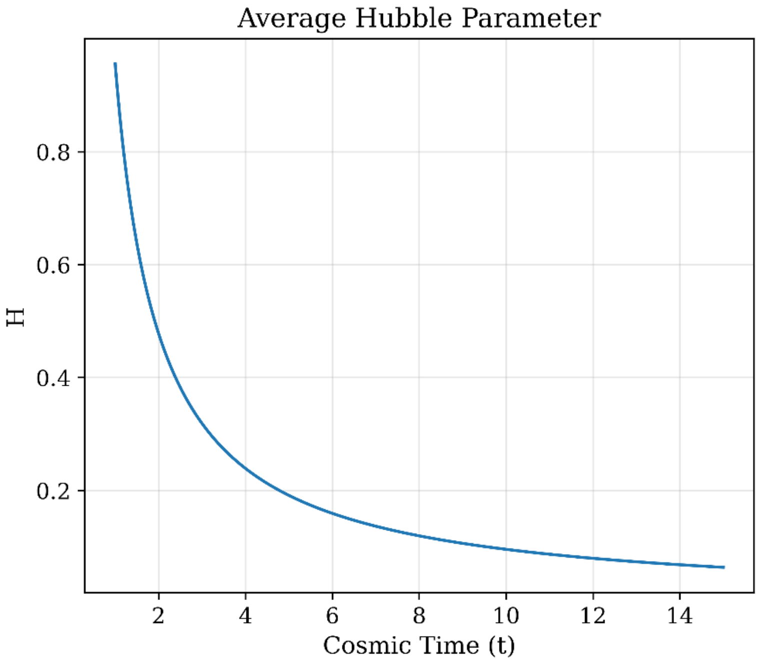

Figure 4: Expansion Rate (H) and Deceleration (q) *Discussion:* The Hubble parameter shows a decreasing expansion rate as the universe ages. The deceleration parameter q remains in the positive regime for the dust filled model, but the repulsive nature of the C-field prevents the expansion from halting entirely.

Conclusion

In this research, we have successfully modeled a Bianchi Type-III dust filled universe within the framework of C-field theory. The investigation reveals that the introduction of a creation field fundamentally alters the singularity and density evolution of the universe. We have demonstrated that the off-diagonal constraint of the Bianchi Type-III metric forces an equality between two of the scale factors ($A = B$), yet preserves a high degree of anisotropy compared to the z-axis.

Mathematically, the energy density in this model does not follow the standard $1/R^3$ dilution law found in Friedmann models. Instead, the creation field acts as a continuous source of energy, providing a repulsive gravitational effect that drives expansion even in a zero-pressure dust environment. The graphical representations further confirm that while the expansion rate H slows over time, the matter density remains significant due to the creation process.

Ultimately, this model suggests that alternative theories of gravitation, specifically those involving creation fields, provide a viable theoretical path for explaining the large-scale structure of the universe without the need for a singular Big Bang. This work contributes to the broader understanding of inhomogeneous and anisotropic cosmological solutions and serves as a foundation for further studies into more complex fluids or dark energy interactions within C-field theory.

Declarations: -

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Ethics approval Not applicable.

Conflict of Interest The authors have no relevant financial or non-financial interests to disclose.
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